Rotation in R^3

Rotation in R^3 (1 of 3)

Here you will see how linear transformations can be used to rotate figures in R^3.

Say you want to rotate the point (x, y, z) counterclockwise about the z-axis through an angle θ, as shown in Figure 6.6

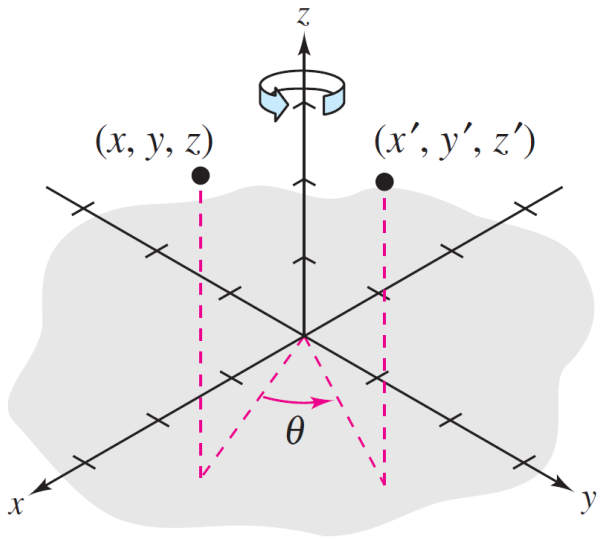

Figure 6.6

Rotation in R^3 (2 of 3)

Letting the coordinates of the rotated point be (x', y', z'), you have

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \\ z \end{bmatrix}.$$

Rotation in R^3 (3 of 3)

Example 4 uses matrices to perform rotations about the *z*-axis. Similarly, you can use matrices to rotate figures about the *x*- or *y*-axis.

Rotation About the *x*-Axis

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

Rotation About the *y*-Axis

$$\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

Rotation About the *z*-Axis

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$